

Question 1 - Mathematical Induction

Show that $3^{3n-2} + 2^{3n+1}$ is divisible by 19 for all positive integers $n = 1, 2, \dots$

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Question 2 - Complex numbers

Find all the complex solutions of $e^z = -2$.

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Question 3 - Limits

Find values of a and b such that $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-3}{x} = 1$.

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Question 4 - Limits

Prove, using the precise definition of limit, that $\lim_{x \rightarrow -1} (2x + 1) = -1$.

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Question 5 - Mean Value Theorem

Prove that for all real numbers a and b , the inequality $|\sin(a) - \sin(b)| \leq |a - b|$ holds.

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Question 6 - Differentiation

Obtain the derivative of $y = (3x)^{\sin(\sqrt{x})+2x}$.

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Question 7 - L'Hospital's Rule

Find all integer values of n for which the limit $\lim_{x \rightarrow \infty} x^n e^{-x}$ is well defined.

[Notes: Recall that well-defined means that the Limit is finite.]

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Question 8 - Fundamental Theorem of Calculus

Find $\frac{d^2}{dx^2} \int_0^x \left(\int_1^{\sin t} \sqrt{1+u^4} du \right) dt$.

[**Hints:** apply the Fundamental Theorem of Calculus 2 times. It may be useful to first regard the integral $\int_1^{\sin t} \sqrt{1+u^4} du$ as a function $f(t)$. That is, first define $f(t) = \int_1^{\sin t} \sqrt{1+u^4} du$ and use the FTC to compute $\frac{d}{dx} \int_0^x f(t) dt$. Then, proceed with the computation of the derivative of what you obtained.]

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Question 9 - Integration

Find the integral $\int \frac{dx}{ax^n+bx}$, where a and b are non-zero real constants and n is any positive integer greater than 1.

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Question 10 - Integration

Compute the integral $\int \sec^2 \theta \ln(\tan \theta) d\theta$.

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Question 11 - Application of integration

Find the area of the surface of revolution obtained by revolving the curve defined by $y^2 + 4x = 2 \ln y$, from $y = 1$ to $y = 2$, around the x -axis.

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Question 12 - Taylor series

Compute (or approximate) the integral $\int \sqrt[a]{x} \cdot e^{x^b} dx$ for arbitrary positive integers a and b .

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Question 13 - Differential Equations

Solve the differential equation $y' \cos x + y \sin x = 1$

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Question 14 - Differential Equations

Solve the differential Equation $x^4 y^2 y' = (y^3 - 1)^{3/5}$

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